

Day 2: Advanced Sequences & Calendar Logic Worksheet

Topic: Large-term sequence rules, digit strings, and modular calendar arithmetic

Worked Examples & Core Strategies

1. Finding Large Terms in Periodic Sequences

Pattern: $A, B, C, D, A, B, C, D, \dots$ (Period length = 4)

To find the 2009th term, compute $2009 \pmod{4}$:

$$2009 = 4 \times 502 + 1 \implies \text{Remainder } 1 \implies \text{1st position in period} = A.$$

2. Calendar Days of the Week (Mod 7 Arithmetic) If today is Saturday, what day of the week will it be in 100 days?

$$100 \div 7 = 14 \text{ full weeks with a remainder of } 2 \text{ days.}$$

Count 2 days forward from Saturday: **Monday**.

Part I: Number & Digit Sequences

Problem 1: Consider the Fibonacci sequence: $1, 1, 2, 3, 5, 8, 13, \dots$. What is the **sum of the next 3 terms**?

- (A) 95
- (B) 104
- (C) 110
- (D) 118

Problem 2: Oscar points to five fingers one by one repeatedly in order: First, Second, Third, Fourth, Fifth, Fourth, Third, Second, First, Second, ... Which finger will he point to on his **2009th** count?

- (A) First finger
- (B) Second finger
- (C) Third finger
- (D) Fourth finger
- (E) Fifth finger

Problem 3: Andro, Soso, Gia, and Niko play catch in a circle, passing the ball in that exact order continuously. If Andro currently holds the ball, who will have the ball after the **1001st** pass?

- (A) Andro
- (B) Soso
- (C) Gia
- (D) Niko

Problem 4: Salome, Ia, and Lali pass a ball around in a circle in that order. Salome starts with the ball. Right after the girls complete their 6th pass, Ira joins the game. Where should Ira stand so that the ball lands on her after the **199th pass** and after the **200th pass**?

- (A) Between Salome and Ia
- (B) Between Ia and Lali
- (C) Between Lali and Salome
- (D) Immediately after Salome only

Problem 5: The number 16 is written on a whiteboard. Every minute, the number on the board is erased and replaced by a new number equal to: (product of its digits) + 12. What number will be written on the whiteboard in **exactly 1 hour** (60 steps)?

- (A) 16
- (B) 18
- (C) 20
- (D) 24

Problem 6: A binary sequence is formed by expanding blocks of ones and zeros as follows:

10110011100011110000...

What digit is located at the **199th position** of this sequence?

- (A) 0
- (B) 1
- (C) 2
- (D) Cannot be determined

Problem 7: The natural numbers are written consecutively in a single continuous row:

12345678910111213...

- a) What digit occupies the **100th position**?
- b) What digit occupies the **1000th position**?

- (A) a) 5; b) 3
- (B) a) 5; b) 2
- (C) a) 0; b) 3
- (D) a) 1; b) 6

Part II: Calendar Days & Dates

Problem 8: Today is Sunday. What day of the week will it be **90 days from today**?

- (A) Friday
- (B) Saturday
- (C) Sunday
- (D) Monday

Problem 9: Today (Sunday) is Nika's birthday. What day of the week is Giorgi's birthday if he is **50 days younger** than Nika?

- (A) Friday
- (B) Saturday
- (C) Monday
- (D) Tuesday

Problem 10: Today is Saturday. What day of the week will it be **100 days from today**?

- (A) Sunday
- (B) Monday
- (C) Tuesday
- (D) Wednesday

Problem 11: Today is Sunday. What day of the week will it be:

- a) In 20 days?
 - b) In 200 days?
 - c) In 1 year (365 days)?
- (A) a) Saturday; b) Tuesday; c) Monday
(B) a) Friday; b) Wednesday; c) Monday
(C) a) Saturday; b) Monday; c) Monday
(D) a) Sunday; b) Tuesday; c) Tuesday

Problem 12: Today is Saturday. What day of the week was it:

- a) 10 days ago?
 - b) 100 days ago?
 - c) 1 year (365 days) ago?
- (A) a) Wednesday; b) Thursday; c) Friday
(B) a) Tuesday; b) Thursday; c) Friday
(C) a) Wednesday; b) Friday; c) Friday
(D) a) Tuesday; b) Wednesday; c) Saturday

Problem 13: In the year 1983, there were exactly 53 Saturdays. What day of the week was **December 31st** of that year?

- (A) Friday
- (B) Saturday
- (C) Sunday
- (D) Monday

Problem 14: Nika stated: *”Ereyesterday (the day before yesterday) I was 10 years old, but next year I will turn 13.”* On what date is Nika’s birthday?

- (A) December 30
- (B) December 31
- (C) January 1
- (D) January 2

Problem 15: Dato's grandmother always celebrates her birthday on the exact date she was born. In the year 2000, she celebrated her 16th birthday. On what exact date was grandmother born?

- (A) January 1
- (B) February 28
- (C) February 29
- (D) March 1

Problem 16: In a certain month, exactly three Sundays fell on even-numbered calendar dates. What day of the week was the **22nd** day of that month?

- (A) Friday
- (B) Saturday
- (C) Sunday
- (D) Monday

Problem 17: It is known that 1988 was a leap year. In 1989, there were fewer Wednesdays than Sundays. In the year 1991, were there more Wednesdays or Saturdays?

- (A) More Wednesdays
- (B) More Saturdays
- (C) Equal amount of Wednesdays and Saturdays
- (D) Depends on leap status

Problem 18: In a certain month, there are more Mondays than Tuesdays, and more Sundays than Saturdays. What day of the week was the **5th** day of this month? Can this month be October?

- (A) Friday; No, it cannot be October
- (B) Thursday; Yes, it can be October
- (C) Sunday; No, it cannot be October
- (D) Friday; Yes, it can be October

Problem 19: In the month of December, three Sundays fell on even dates. What day of the week will the **17th** of December be?

- (A) Tuesday
- (B) Wednesday
- (C) Thursday
- (D) Friday

Problem 20: Sopho decided to label days in a recurring pattern: after every 4th day, she restarts counting at 1 (1, 2, 3, 4, 1, 2, 3, 4, 1, ...). On one day, her assigned custom number matched the regular calendar date, both being 3. Exactly one month later, the exact same match occurred (calendar date 3 had custom label 3). When can this match occur again next?

- (A) Exactly 2 months later

- (B) Exactly 3 months later
- (C) Exactly 5 months later
- (D) Exactly 7 months later