

Day 4: Order of Operations, Bracket Rules, and Linear Equations

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Theory

I. Properties of Operations and Bracket Rules. When manipulating algebraic expressions or evaluating arithmetic expressions involving brackets, the following fundamental properties apply:

- **Commutative Properties:**

$$a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a$$

- **Associative Properties:**

$$(a + b) + c = a + (b + c) \quad \text{and} \quad (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

- **Distributive Properties (Expanding Brackets):**

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

$$(a + b)c = ac + bc \quad \text{and} \quad (a - b)c = ac - bc$$

II. Order of Operations (PEMDAS / BODMAS). When evaluating an expression containing multiple operations and nested grouping symbols, operations must be performed in the following strict order:

1. **Grouping Symbols (Brackets):** Evaluate expressions inside grouping symbols from the innermost level to the outermost level in the sequence:

$$\text{Parentheses } (\dots) \longrightarrow \text{Brackets } [\dots] \longrightarrow \text{Braces } \{\dots\}$$

2. **Exponents (Powers):** Compute all exponents and powers.
3. **Multiplication and Division:** Perform all multiplications and divisions in order from left to right.
4. **Addition and Subtraction:** Perform all additions and subtractions in order from left to right.

III. Solving Equations in One Variable. An equation in one variable (such as x) is an algebraic equality. Solving the equation means finding all values of x that make the equality true.

To solve a linear equation with one variable step-by-step:

- Step 1. Expand Brackets:** Use the distributive property $a(b \pm c) = ab \pm ac$ to remove all parentheses, brackets, or braces.
- Step 2. Combine Like Terms:** Simplify both sides of the equation independently by combining variable terms with variable terms and constant terms with constant terms.
- Step 3. Isolate the Variable Term:** Move all terms containing the variable x to one side (usually the left) and all constant numerical terms to the opposite side by performing inverse operations (addition/subtraction).
- Step 4. Solve for x :** Divide or multiply both sides by the coefficient of x to isolate x completely.

Example. Solve $3(2x + 1) + 2(4x + 2) = 35$.

Solution.

$3(2x) + 3(1) + 2(4x) + 2(2) = 35$	(Expand brackets)
$6x + 3 + 8x + 4 = 35$	(Multiply terms)
$14x + 7 = 35$	(Combine like terms)
$14x = 35 - 7$	(Subtract 7 from both sides)
$14x = 28$	(Simplify)
$x = 2$	(Divide by 14)

Order of Operations Practice

Evaluate each of the following arithmetic expressions:

- $6^2 - (32 - 29)^2$
- $[(21 - 17) \times (14 - 9)]^2$
- $11^2 - 3 \times 8 - 5^2 - 19$
- $11^2 - \{3 \times [8 - (5^2 - 19)]\}$
- $11^2 - [3 \times (8 - 5)^2] - 19$
- $12 \times (3 + 11 + 6) \div 2^3$
- $4^2 + 2^2 \times [40 \div (31 - 29)^2]$
- $1^3 \times \{9 + [(9 - 21 + 7) - (2^3 - 5)]\}$

9. $10^2 \times 9 - (4^3 - 4 \times 10)$
10. $4^2 + 3^2 - \{25 - [2 \times (11 + 9 - 13)]\}$
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Basic Equations Practice

Solve each of the following one-variable equations for x :

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1. $x \div 7 - 2 = 5$
 2. $4x + 4 = 32$
 3. $9x - 11 = 16$
 4. $2x + 9 = 27$
 5. $4x + 7 = 63$
 6. $x \div 11 + 4 = 9$
 7. $x \div 8 + 2 = 7$
 8. $x \div 9 + 13 = 21$
 9. $9x - 8 = 64$
 10. $5x + 16 = 46$
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Equations with Brackets

Question 1 — Basic Bracket Equations

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- a) $2(y + 2) = 8$
- b) $3(a + 2) = 12$
- c) $4(x - 4) = 40$
- d) $5(c - 3) = 35$
- e) $7(3 + a) = 49$
- f) $6(6 + p) = 48$

g) $9(x + 4) = 18$

h) $8(3 + m) = 8$

Question 2 — Multi-Step Bracket Equations

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a) $2(4x + 8) = 32$

b) $5(7x - 4) = 15$

c) $5(5x - 3) = 35$

d) $4(6x + 3) = 36$

e) $7(2x - 7) = 7$

f) $6(2x - 3) = 42$

Question 3 — Expanding and Combining Like Terms

a) $3(2x + 1) + 2(4x + 2) = 35$

b) $2(x + 3) + 3(x + 1) = 24$

c) $4(3x - 2) + 8(x + 1) = 100$

d) $6(x + 2) + 4(3 - x) = 30$

e) $5(2x + 3) + 2(5x + 1) = 37$

Question 4 — Equations with Multiple Expanding Terms

a) $2(3x + 1) - 3(x + 2) = 2$

b) $5(2x + 3) - 3(3x - 2) = 22$

c) $4(2x - 3) + 2(x - 4) = 10$

d) $3(4x - 2) + 5(x - 1) = 23$

e) $8(2x - 1) - 3(5x - 3) = 1$

Challenge Problems

C1. Solve for x :

$$2\left(3\left(2(x+1)+4\right)+5\right)=70$$

C2. Solve for x :

$$5\left(x+2\left(x+3(x+1)\right)\right)+4=184$$

C3. Solve for x :

$$12(3x-5)-4(7x-11)+3(2x+1)=93$$

C4. Find the positive integer x satisfying:

$$3\left(4x+5\left(3x-2(x-1)\right)\right)=141$$

C5. Solve the deeply nested equation for x :

$$100-3\left\{40-2\left[15-(2x+1)\right]\right\}=34$$