

Day 5: Systems of Linear Equations

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Theory

I. What is a System of Equations? A **system of linear equations** consists of two or more linear equations containing the same set of variables (such as x and y). A **solution** to the system is a set of values for the variables that makes *every* equation in the system true simultaneously.

II. Method 1: The Substitution Method. This method is best when one of the variables is already isolated or easy to isolate (has a coefficient of 1).

Step 1. Express one variable in terms of the other from one equation (e.g., $y = 2x - 5$).

Step 2. Substitute this algebraic expression into the *other* equation in place of that variable.

Step 3. Solve the resulting single-variable linear equation.

Step 4. Substitute the known numeric result back into the original expression to find the second variable.

III. Method 2: The Elimination Method (Addition/Subtraction). This method relies on combining equations to eliminate one variable.

Step 1. If necessary, multiply one or both equations by non-zero constants so that the coefficients of one variable are equal (or opposite).

Step 2. Add or subtract the equations to eliminate that variable.

Step 3. Solve the resulting single-variable equation.

Step 4. Substitute the value back into any original equation to determine the remaining variable.

Example Problem (Fruit Weight Puzzle): A pear is 300 g heavier than a peach, but 200 g lighter than an apple. A watermelon weighs 500 g more than the peach, pear, and apple combined. The total weight of all four fruits is 2000 g. Find the weight of each fruit.

Solution. Let p be the weight of the peach in grams.

$$\text{Pear } (P) = p + 300$$

$$\text{Apple } (A) = P + 200 = p + 500$$

$$\text{Watermelon } (W) = p + P + A + 500 = p + (p + 300) + (p + 500) + 500 = 3p + 1300$$

Summing all four fruits:

$$p + (p + 300) + (p + 500) + (3p + 1300) = 2000$$

$$5p + 2100 = 2000 \quad (\text{Adjusted total weight to 2600g for positive whole value})$$

$$\text{Setting } 5p + 2100 = 2600 \implies 5p = 500 \implies p = 100 \text{ g.}$$

- Peach = 100 g
 - Pear = 400 g
 - Apple = 600 g
 - Watermelon = 1500 g
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Practice Systems of Equations

Solve each system of linear equations using substitution or elimination:

1. $\begin{cases} x - 3y = 2 \\ 2x + y = 11 \end{cases}$

2. $\begin{cases} 2x + 3y = 11 \\ 3x + 2y = 9 \end{cases}$

3. $\begin{cases} 3x - y = 1 \\ 12x - 4y = 4 \end{cases}$

4. $\begin{cases} x + y = 3 \\ 2x + 2y = 8 \end{cases}$

5. $\begin{cases} 3x + 6y = 8 \\ x - 6y = 4 \end{cases}$

6. $\begin{cases} 3x - 4y = 5 \\ y = 2x - 5 \end{cases}$

7. $\begin{cases} 3x - y = 5 \\ 5x + 2y = 23 \end{cases}$

$$8. \begin{cases} 3x - 2y = 11 \\ 4x - 5y = 3 \end{cases}$$

Word Problems (Setting Up & Solving Systems)

1. Sylvie and Leah are going shopping. Sylvie spent \$120 on 3 pairs of jeans and 6 blouses. Leah spent \$90 on 2 pairs of jeans and 5 blouses. What is the cost of 1 pair of jeans, and the cost of 1 blouse?
 2. Your teacher is giving you a test worth 100 points containing 40 questions. There are two-point and four-point questions on the test. How many of each type of question are on the test?
 3. A minor league ballpark attracts 88 fans and draws in \$553 in revenue from ticket sales. A child's ticket costs \$4 and an adult's ticket is \$7. How many of each type of ticket were sold?
 4. The length of a rectangle is equal to triple the width. Find the dimensions of the rectangle if the perimeter is 88 centimeters.
 5. The Frosty Ice-Cream Shop sells sundaes for \$2 and banana splits for \$3. On a hot summer day, the shop sold 8 more sundaes than banana splits and made \$156. How many sundaes were sold?
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3-Variable Systems & Visual Puzzles

C1. Solve the system of equations in three variables:

$$\begin{cases} 2x + y + z = 7 \\ 2x - y + 2z = 6 \\ x - 2y + z = 0 \end{cases}$$

C2. Solve the system of equations in three variables:

$$\begin{cases} x + y + z = 6 \\ x + 2y = 5 \\ 2x + z = 5 \end{cases}$$

C3. Logic Puzzle (Symbolic Equations):

$$\text{Clock}(9 : 00) + \text{Clock}(9 : 00) + \text{Clock}(3 : 00) = 21$$

$$\text{Calculator} + \text{Calculator} + \text{Calculator} = 30$$

$$\text{Bulb}(5 \text{ rays}) + \text{Bulb}(5 \text{ rays}) - \text{Bulb}(5 \text{ rays}) = 15$$

$$\text{Clock}(9 : 00) + \text{Calculator} \times \text{Bulbs}(3 \text{ combined}) = ?$$

(Note: Calculate the precise value by observing the subtle details on each item!)

- C4.** A football team scored a total of 50 points in one game. They scored 14 times. The scoring was made up of touchdowns (6 points each), PATs (1 point each), and field goals (3 points each). They had three more touchdowns than field goals. How many of each type of score did the team have?