

Number Theory & Calendar Logic Problem Set

Olympiad Mathematics Practice

Part I: Number Theory Problems

1. How many tens will we get if we add 3 hundreds to 6 thousands, and then divide the resulting sum by 9 tens?
2. Replace the asterisks with digits to make each equality correct:

$$(a) \begin{array}{r} 5 * \\ + * 84 \\ \hline ***0 \end{array}$$

$$(b) \begin{array}{r} 6 5 * \\ + * 0 4 \\ \hline **8 \end{array}$$

$$(c) \begin{array}{r} * * * * \\ - \\ \hline 2856 \end{array}$$

$$(d) \begin{array}{r} 3 * 6 \\ + * 2 * \\ \hline 804 \end{array}$$

3. What is the last digit of the value of the following expression?

$$199 \cdot 197 \cdot 195 + 192 \cdot 193 \cdot 194$$

4. What digit can the product of four consecutive natural numbers end with?
5. Find:
 - (a) The smallest four-digit number whose product of digits is equal to 30.
 - (b) The largest four-digit number whose product of digits is equal to 30.
6. Find the smallest natural number that is divisible by 18 and has a digit sum of 18.
7. What digit must replace A so that the number $52A71$ is divisible by 9?
8. What digit should be placed in the expression $98765 - 4321*$ in place of the asterisk so that the difference is divisible by 9?
9. Replace the asterisks in the notation $4 * 13*$ with digits so that the resulting number is divisible by 15.
10. Replace the asterisks in $42 * 4*$ with digits so that the number is divisible by 36. In how many ways can this be done?
11. Given a five-digit number $46a7b$ where all digits are distinct from one another, find all ordered pairs (a, b) for which the number is divisible by 45.

12. Given consecutive natural numbers $1, 2, 3, \dots, n$, exactly 63 of which are multiples of 17. Find the largest possible value of n .
13. The product of 199 natural numbers is equal to 199. What is their sum equal to?
14. A two-digit number is multiplied by 6. Then 9 is added to the product, resulting in a three-digit number composed of three identical digits. Find all such two-digit numbers.
15. The sum of the digits of the birth years of Levani and his younger brother Giorgi matched their respective ages in the year 2017. How old was Levani in 2017?
16. Vaniko answered a question posed in class: the answer was a prime number, the sum of the ages of Giorgi and Luka was an even number, while Elene was 15 years old. It turned out that among the girls, only one statement was true, and among the boys, only one statement was true. What was the answer to the problem?
17. Find the smallest natural number that leaves a remainder of 6 when divided by 8, and a remainder of 1 when divided by 6.
18. Find the largest three-digit number that leaves a remainder of 7 when divided by 11, and a remainder of 3 when divided by 7.
19. Find the largest whole number which, when divided by 10 with a remainder, yields a quotient equal to 57.
20. Find the smallest natural number greater than 1 that leaves a remainder of 1 when divided by both 9 and 12.
21. Find the smallest natural number that leaves a remainder of 1 when divided by 3, a remainder of 2 when divided by 4, and a remainder of 3 when divided by 5.

Part II: Advanced Calendar Logic Problems

22. In a non-leap year, a specific day of the week appears 53 times. If January 1st of this year was a Thursday, on what day of the week will October 31st fall in the subsequent year (which is a leap year)?
- (A) Saturday
 - (B) Sunday
 - (C) Monday
 - (D) Tuesday
23. In a certain month, three Tuesdays fell on even-numbered calendar dates. What day of the week was the 18th day of that month?
- (A) Friday
 - (B) Saturday
 - (C) Sunday
 - (D) Wednesday
24. A leap year contains 366 days. What is the maximum possible number of Sundays that can occur within any consecutive 100-day window during a single leap year?
- (A) 14
 - (B) 15
 - (C) 16
 - (D) 17
25. Alex notes that his birthday in year N fell on a Monday. In year $N + 2$, his birthday fell on a Wednesday. In year $N + 4$, his birthday fell on a Friday. Which of the following statements **MUST** be true about the leap status of these years?
- (A) Year $N + 1$ was a leap year.
 - (B) Year $N + 2$ was a leap year.
 - (C) Year $N + 3$ was a leap year.
 - (D) None of the years N through $N + 4$ contained a February 29th across his birth dates.
26. In a standard non-leap year of 365 days, how many months have more Thursdays than Fridays if the year starts on a Wednesday?
- (A) 3
 - (B) 4
 - (C) 5
 - (D) 7