

Day 2: Exponents and Prime/Composite Numbers

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Theory

Natural Exponents. The power a^n is defined as the n -th natural power of the number a . By definition,

$$a^n = \underbrace{a \cdot a \cdots a}_{n \text{ times}},$$

where a^n is the product of n factors, each equal to a . The number a is called the *base*, and n is called the *exponent*. In particular, we define $a^1 = a$. If $a \neq 0$, then it is convention that $a^0 = 1$. Note that 0^0 is undefined.

V. Prime and Composite Numbers. A natural number is called *prime* if it has exactly two distinct natural divisors: 1 and the number itself. For example, 2, 3, 5, 7, 11, 13 are prime numbers. 15 is not a prime number because it has four natural divisors: 1, 3, 5, and 15. The number 1 has only one natural divisor. A natural number greater than 1 that is not prime is called *composite*. For composite numbers, the following holds:

Theorem. Any composite natural number can be uniquely expressed as a product of prime factors.

Example. $14 = 2 \cdot 7$, $81 = 3 \cdot 3 \cdot 3 \cdot 3$, $484 = 2 \cdot 2 \cdot 11 \cdot 11$.

When factoring a composite number into prime factors step-by-step:

$$\begin{array}{r|l} 525 & 3 \\ 175 & 5 \\ 35 & 5 \\ 7 & 7 \\ 1 & \end{array}$$

As a result, we obtain $525 = 3 \cdot 5 \cdot 5 \cdot 7 = 3 \cdot 5^2 \cdot 7$.

Textbook Problems

1. (1.1) Factor into prime factors:

88; 96; 124; 144; 460; 588; 1140; 2400.

2. (1.8) Find the smallest prime divisor of:

- 1) 24;
- 2) 77;
- 3) 128;
- 4) 4128.

3. (1.9) Find the largest prime divisor of:

- 1) 28;
- 2) 99;
- 3) 200;
- 4) 532.

4. (1.10) Find the number of distinct prime divisors of the following numbers:

- 1) 12;
- 2) 13;
- 3) 50;
- 4) 120.

5. (1.11) What is the sum of the prime divisors of 66?

6. (1.12) Find the total number of natural divisors for:

- 1) 15;
- 2) 19;
- 3) 27;
- 4) 48.

7. (12) How many natural divisors does 36 have?

- a) 4 b) 5 c) 8 d) 9

8. (13) How many prime divisors does 36 have?

- a) 1 b) 2 c) 3 d) 4

Additional Practice Problems

- A1.** Factor the following numbers into prime factors: 180, 504, 1080, 2520.
- A2.** Find the smallest natural number n such that $180 \cdot n$ is a perfect square.
- A3.** Compare the numbers 2^{60} and 3^{40} .
- A4.** How many trailing zeros does the number $2^3 \cdot 3^2 \cdot 5^2$ end with?
- A5.** How many trailing zeros does the number $2^5 \cdot 3^2 \cdot 5^7 \cdot 7^1$ end with?
- A6.** Find the total number of natural divisors of 36.
- A7.** Find the total number of odd natural divisors of 18.
- A8.** Find the sum of all natural divisors of 30.
- A9.** A natural number $N = 2^a \cdot 3^b$ has exactly 12 natural divisors. Find the minimum possible value of N , given that $a \geq 1$ and $b \geq 1$.