

Day 6: Base-10 Representation & Digits Problems

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Theory: Positional Base-10 Systems

I. Decimal Expansion of Numbers. In the base-10 system, the position of each digit determines its value:

- A two-digit number with tens digit a and units digit b is written as $\overline{ab} = 10a + b$.
- Reversing its digits yields $\overline{ba} = 10b + a$.
- A three-digit number with digits a, b, c is represented as $\overline{abc} = 100a + 10b + c$.

II. Appending Digits (Positional Shifts).

- **Appending to the right:** If a digit k is written to the right of a number x , the original number shifts one decimal place to the left:

$$\text{New Number} = 10x + k$$

- **Appending a two-digit number to the right:** If a two-digit number k (e.g., 11) is appended to the right of x :

$$\text{New Number} = 100x + k$$

- **Appending to the left:** If a digit k is written to the left of an n -digit number x :

$$\text{New Number} = k \cdot 10^n + x$$

Example Problem: If the digits of a two-digit number $\overline{ab}$ are interchanged, the sum of the original number and the new number is always divisible by 11. Prove this algebraically.

Proof.

$$\overline{ab} + \overline{ba} = (10a + b) + (10b + a) = 11a + 11b = 11(a + b)$$

Since $11(a + b)$ is a multiple of 11, the sum is always divisible by 11.

Part 1: Two-Digit Problems

1. The sum of the digits of a two-digit number is 12. If the digits are reversed, the new number is 18 greater than the original number. Find the original number.
 2. The sum of the digits of a two-digit number is 11. If we interchange the digits, the new number formed is 45 less than the original. Find the original number.
 3. The sum of the digits of a 2-digit number is 10. The tens digit is 4 times the ones digit. Find the number.
 4. A 2-digit number is 10 times the sum of its digits. The tens digit is 2 greater than the units digit. Find the number.
 5. The sum of the digits of a 2-digit number is 14. If 18 is added to the number, the result is the number with its digits reversed. Find the original number.
 6. The sum of the digits of a 2-digit number is 9. If the digits are reversed, the new number is 9 less than 3 times the original number. Find the original number.
 7. The ones digit in a two-digit number is 7 greater than the tens digit. If we interchange the digits in the number, we obtain a new number that, when added to the original number, results in the sum 99. Find this number.
 8. The tens digit in a two-digit number is 2 greater than the ones digit. If we interchange the digits in the number, we obtain a new number that, when added to the original number, results in the sum 176. Find this number.
 9. The tens digit in a two-digit number is 3 less than the ones digit. If we interchange the digits in the number, we obtain a new number that, when added to the original number, results in the sum 143. Find this number.
 10. The ones digit in a two-digit number is 3 less than the tens digit. If we interchange the digits in the number and add this new number to the original number, the sum is 77. Find this number.
 11. A two-digit number is 8 times the sum of its digits. If 45 is subtracted from the number, the positions of the digits are interchanged. The number is:

(a) 56(b) 83(c) 72(d) None
 12. A two-digit number and the resulting number when the digits are reversed are in the ratio 2 : 9. If the sum of the digits is 9, find the original number.
 13. The units digit of a number exceeds twice its tens digit by 3. If the digits are reversed, the new number is 54 more than the original number. Find the original number.
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Part 2: Three-Digit & Page Numbering Problems

14. The number of digits used in numbering each page of a book of 150 pages is:

- (a) 342 (b) 348 (c) 328 (d) 322

15. A mathematics book contains 20 pages (10 sheets). One sheet is missing. The sum of the page numbers of the remaining pages is 195. Which two page numbers were written on the missing sheet?

- (a) 9, 10 (b) 11, 12 (c) 5, 6 (d) 7, 8

16. The sum of the digits of a three-digit number is 15. The tens digit is less than the units digit by 3. If the digits are reversed, the new number diminished by 78 is three times the original number. Find the original number.

17. The sum of the digits of a three-digit number is 15. The tens digit is less than the units digit by 2. If the digits are interchanged, the new number decreased by 7 is twice the original number. Find the original number.

18. The hundreds digit of a three-digit number is the sum of the tens and units digits, and the units digit exceeds the tens digit by 2. Find the number if it is 52 times the sum of its digits.

Part 3: Digits Appended to Left or Right

A1. A natural number x ends with the digit 2. If this last digit 2 is removed (or moved to the far left), or if we append the two-digit number 11 to the right of x , write an algebraic expression for the resulting number.

- (a) $x + 11$ (c) $11x$ (e) $100x + 11$
(b) $10x + 11$ (d) $x + 110$

A2. A single digit 7 is written to the right of a natural number x . Express the new number in terms of x , and determine how much larger the new number is compared to x .

A3. A digit 3 is written to the left of a three-digit natural number y . If the resulting four-digit number is 9 times the original number y , find y .