

Day 7: Fractions, Decimals & Rates

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September 2026

Theory: Operations & Fraction Properties

I. Comparison of Fractions.

- **Same Denominator:** $\frac{a}{c} < \frac{b}{c}$ if $a < b$.
- **Same Numerator:** $\frac{a}{b} > \frac{a}{c}$ if $b < c$ (for positive a, b, c).
- **Cross-Multiplication:** $\frac{a}{b} \geq \frac{c}{d} \iff a \cdot d \geq b \cdot c$.

II. Arithmetic Operations.

- **Addition/Subtraction:** Find a common denominator: $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$.
- **Multiplication:** Multiply straight across: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.
- **Division:** Multiply by the reciprocal: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

III. Converting Decimals to Fractions. To convert $0.\overline{a_1a_2 \dots a_k}$ to a fraction, set $x = 0.\overline{a_1a_2 \dots a_k}$, multiply by 10^k , and solve for x :

$$10^k x = a_1a_2 \dots a_k \implies x = \frac{a_1a_2 \dots a_k}{10^k}$$

Example Problem (Complex Fraction Simplified): Evaluate $S = \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{n}\right)$.
Solution. Express each term as a single improper fraction:

$$S = \left(\frac{1}{2}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{3}{4}\right) \dots \left(\frac{n-1}{n}\right)$$

Notice that every intermediate numerator cancels with the preceding denominator:

$$S = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \dots \frac{n-1}{n} = \frac{1}{n}$$

Part 1: Basic Operations & Comparison

1. Compare the following fraction pairs without converting to decimals:

(a) $\frac{4}{11}$ and $\frac{6}{11}$

(b) $\frac{11}{15}$ and $\frac{27}{31}$

2. Determine which fraction in each pair is larger and closer to 1:

(a) $\frac{5}{6}$ or $\frac{7}{8}$

(b) $\frac{127}{130}$ or $\frac{10}{13}$

3. Arrange the following numbers in ascending order:

$$\frac{1}{24}, \quad \frac{11}{24}, \quad \frac{12}{24}, \quad \frac{15}{24}, \quad \frac{16}{24}, \quad \frac{22}{24}, \quad \frac{3}{12}, \quad \frac{5}{12}, \quad \frac{3}{6}, \quad \frac{1}{4}, \quad \frac{2}{3}, \quad \frac{1}{2}$$

4. Evaluate the arithmetic expressions:

(a) $3\frac{3}{4} + 1\frac{5}{12}$

(c) $3\frac{2}{3} \cdot 5\frac{5}{11}$

(b) $7\frac{1}{3} - 2\frac{4}{5}$

(d) $4\frac{4}{9} \div 2\frac{2}{9}$

Part 2: Multi-Step Multi-Operation Expressions

5. Compute the value of the expressions:

(a) $(7\frac{1}{2} + 2\frac{1}{4}) \cdot 3 + (5\frac{1}{2} + 2\frac{2}{3}) \cdot 100$

(b) $32 \cdot \frac{12}{5} - \frac{1}{3}$

(c) $\frac{3}{5} \div \frac{7}{40} - 1\frac{1}{10} \cdot 1\frac{5}{11}$

6. Evaluate the nested arithmetic expressions:

(a) $(6\frac{1}{5} - 1\frac{3}{10}) \cdot 6 + \frac{7}{20} \cdot 20 - 12 \div \frac{6}{7} + 8 \cdot \frac{3}{4}$

(b) $(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) \dots (1 - \frac{1}{24}) \cdot 24$

7. Calculate the complex structured fractions:

(a) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}$

(b) $2 - \frac{1}{2 - \frac{1}{2 - \frac{2}{3}}}$

Part 3: Decimals & Exponents

8. Express each fraction as a terminating decimal:

(a) $\frac{3}{5}$

(b) $\frac{7}{8}$

(c) $\frac{2}{50}$

(d) $\frac{7}{35}$

9. Convert each decimal to an exact fraction in simplest form:

(a) 0.375

(b) 0.23

(c) 1.16

(d) 3.125

10. Find the 187th digit after the decimal point in the decimal representation of $\frac{2}{7}$.

11. Compute the precise decimal values:

(a) $(0.001)^2$

(c) $\frac{(0.03)^2}{0.003}$

(b) $\frac{0.05}{(0.01)^2}$

(d) $\frac{0.8}{(0.02)^3}$

12. Compute the value of t :

If $t = \frac{1}{2^9 \cdot 5^3}$, find how many zeros appear immediately after the decimal point before the first non-zero digit.

13. $\left(\frac{2}{15} + 1\frac{7}{12}\right) \cdot \frac{30}{103} - \left(2 : 2\frac{1}{4}\right) \cdot \frac{9}{32};$

14. $\frac{\left(\frac{5}{6} : 30 + \frac{35}{36}\right) \cdot \left(3\frac{1}{2} + 2\right)}{2.7 : 0.3 + \frac{4}{9} \cdot 4.5};$

15. $\frac{2\frac{5}{8} \cdot \frac{2}{3} - 2\frac{5}{12}}{\left(3\frac{1}{12} + 4.375\right) : 19\frac{8}{9}};$

16. $1\frac{7}{20} : 2.7 + 2.7 : 1.35 + \left(0.4 : 2\frac{1}{2}\right) \cdot \left(4.2 - 1\frac{3}{40}\right);$

17. $\frac{3 : \frac{2}{5} - 0.09 : \left(0.15 : 2\frac{1}{2}\right)}{0.32 \cdot 6 + 0.03 - (5.3 - 3.88) + 0.67};$

18. $(0.8 \cdot 7 + 0.64) \left(1.25 \cdot 7 - \frac{4}{5} \cdot 1.25\right) + 31.64;$

19. $\left(\frac{0.216}{0.15} + \frac{2}{3} : \frac{4}{15}\right) + \left(\frac{196}{225} - \frac{7.7}{24\frac{3}{4}}\right) + 0.695 : 1.39;$

20. $1.7 : \frac{\left(4.5 \cdot 1\frac{2}{3} + 3.75\right) \cdot \frac{7}{135}}{\frac{5}{9}} - \left(0.5 + \frac{1}{3} - \frac{5}{12}\right);$

Part 4: Word Problems, Speeds & Rates

13. A tourist walked 21 km in 2 hours, while a second tourist walked 28 km in 3 hours. Which tourist walked faster, and by how many kilometers per hour?

14. Gia, Dato, and Beka painted a fence together. Gia finished $\frac{3}{10}$ of the total work, Dato completed $\frac{5}{8}$, and Beka finished $\frac{3}{40}$. Who completed the largest portion of the work?

15. A wheat harvest yields $1\frac{2}{5}$ tons per $\frac{2}{5}$ hectares.

- (a) How many tons of wheat are harvested per single hectare?
- (b) How many hectares are required to harvest exactly 1 ton of wheat?

16. Three vehicles depart simultaneously from point A to point D :

- Truck goes through B ($A \rightarrow B \rightarrow D$: 41 km + 30 km) at 40 km/h.
- Bus goes through C ($A \rightarrow C \rightarrow D$: 28 km + 42 km) at 34 km/h.
- Car travels directly along $A \rightarrow D$ (75 km) at 50 km/h.

Which vehicle arrives at point D last, and how long does its journey take?

Part 5: Algebraic Integer Variable Analysis

A1. How many fractions with a denominator of 24 lie strictly between $\frac{2}{3}$ and $\frac{7}{8}$? List all such fractions.

A2. Find all natural numbers x such that the fraction $\frac{x}{7}$ is improper and its value is strictly less than 2.

A3. Find all natural numbers x such that the fraction $\frac{x}{11}$ is proper and its value is strictly greater than $\frac{1}{2}$.

A4. Find the sum of all natural numbers x for which $\frac{x}{11}$ is a proper fraction and $\frac{x}{8}$ is an improper fraction.